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SOME EFFECTS OF FLIGHT PATH AND ATMOSPHERIC VARIATIONS ON THE BOOM PROPAGATED FROM A SUPERSONIC AIRCRAFT

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SUMMARY

Equations for the shock wave envelope and cusp line associated with the boom propagated from a supersonic aircraft are formulated in terms of the moving-trihedral coordinate system for flight in a uniform atmosphere and also in an atmosphere with a linear sound-speed gradient. Ray-tube theory is used to calculate the lateral distribution of boom intensity in an atmosphere with a linear sound-speed gradient and also to investigate the effect of a general wind and sound-speed gradient on the ground-track intensity. The relative effects of wind and temperature gradient are treated. The mechanisms of focusing by winds and by ground structures are discussed qualitatively.

INTRODUCTION

The advent of supersonic aircraft with their attendant noise problems has been accompanied by considerable literature on the subject of sonic-boom propagation in the atmosphere. The location of the ground pattern and the calculation of boom intensity have been treated in a number of papers (ref. 1, for example) for unaccelerated flight in a uniform atmosphere. References 2 and 3 contain an analysis based on geometrical acoustics for a uniform atmosphere of the conditions for focusing resulting from aircraft acceleration or turning maneuvers, together with an estimate of the intensity of the boom at the focal points. Some effects of a linear sound-speed variation with altitude were studied in reference 4. For this simplest kind of atmospheric nonuniformity, the equations for the wave and wave envelope were derived in reference 4; but computing-machine techniques have been required for sound-ray tracing when the effects of winds or of a general sound-speed variation are considered (refs. 5 and 6). The geometry of sound propagation in a nonuniform atmosphere has also been treated in a number of studies in which the sonic-boom problem was not directly involved (for example, refs. 7 and 8).

This paper presents a study of several aspects of the effects of acceleration, turning maneuvers, and refraction on the shock-wave distribution and boom intensity, with some emphasis on the application to flights of supersonic commercial aircraft (ref. 9). In extending previous results for the present analysis, an effort has been made to avoid duplication of those results, within reasonable limits. For example, in reference 2, the conditions for cusp formation

are discussed; but, in the present paper, the actual spatial distribution of the cusp line is treated, and parametric equations for this line are derived. The use of the moving-trihedral coordinate system in this analysis results in some simplification of the equations.

The moving-trihedral method of analysis is also applied to the study of the distribution of the shock envelope and cusp line in an atmosphere with a linear sound-speed gradient for constant-altitude flight. The lateral distribution of boom intensity for flight in this kind of atmosphere is then calculated by means of a ray-tube analysis. The ray-tube method is also used to investigate the effect on the ground-track boom intensity of an arbitrary vertical variation of sound speed and headwinds or tailwinds. The relative effects of wind and temperature gradients on refracting the rays are then treated. In addition, the mechanisms of focusing of slightly inclined rays by winds and by ground structures are discussed qualitatively.

SYMBOLS

ΔA cross-sectional area of ray tube

ΔA_g area of intersection of ray tube with ground

a   sound speed

B   angle between vertical plane containing tangent to flight path and vertical plane containing sound ray under consideration

c   characteristic velocity in Snell's law

Δd perpendicular distance between upper and lower bounding rays of ray tube

g   acceleration due to gravity

$H = 1 - \dfrac{kh}{a_g}$

h   flight altitude, when assumed constant

I   boom overpressure intensity

k   approximate absolute value of vertical sound-speed gradient in ICAO standard atmosphere, 0.00407 sec^{-1}

$\vec{k}$ unit vector in positive z-direction

l wind gradient when variation of wind speed with altitude is linear

M	Mach number
Q	infinitesimal radial coordinate of initial ray cone (fig. 4(b))
R	distance of generic point of wave front at time t from point of emission at time τ (propagation in a uniform atmosphere)
r	distance along ray path
$\vec{r}_N$	unit vector in ray-path direction
s	distance along flight path
T	radius of torsion of flight path
t	time at wave front
$\tilde{t} = t - \tau$	
u,v	components of wind velocity along x- and y-axes
V	airplane airspeed, $ds/d\tau$
x,y,z	rectangular Cartesian coordinates fixed with respect to earth, with z positive upward and equal to zero at ground level
X,Y,Z	rectangular Cartesian coordinates referred to moving trihedral of flight trajectory (fig. 1)
$\cos\alpha$, $\cos\beta$, $\cos\gamma$	direction cosines of wave-front normal
θ	absolute value of inclination angle of wave-front normal
$\lambda = 1 - \dfrac{1}{M^2}$	
ξ, η, ζ	aircraft position coordinates in x,y,z system
ρ	radius of curvature
τ	time at which a disturbance is produced at source
$\Delta\psi$	angle subtended at source by an incremental section of ray cone at source
X	absolute value of angle of inclination of secondary tangent vector
Ω	angle with horizontal measured in plane perpendicular to flight path, positive downward

Subscripts:

c   cuspidal ridge

g   conditions at ground level

gt   ground track

max maximum

o   conditions at flight level

u   conditions in a uniform atmosphere

A prime is used to denote derivative with respect to τ.

ANALYSIS

Formation of Superboom Region Due to
Aircraft Acceleration and Turning

In this section, as throughout the paper, it is assumed that the geometry of sonic-boom propagation is described by the theory of geometric acoustics. This assumption is usually made in the literature, but the methods of geometric acoustics are not applicable quantitatively to the prediction of boom intensities near the aircraft or in the immediate vicinity of a cusp. They are useful for studying the spatial distribution of the shock envelope, for locating cusp lines, and for estimating relative intensities from ray-tube cross-sectional areas.

A second assumption, which is made in this section only, is that the atmosphere is uniform. Aside from the considerable simplification of the equations which results from this approximation, it will be seen that, for the special case of a high supersonic Mach number aircraft flying at or near cruise conditions, the effects of turning or accelerating are not normally greatly altered by the presence of atmospheric nonuniformities.

The X,Y,Z coordinates referred to the moving-trihedral coordinate system (ref. 10) are as shown in figure 1. In this system the equation of a sound wave front at time t due to a disturbance produced at time τ is

$$X^2 + Y^2 + Z^2 = a^2(t - \tau)^2 = R^2 \tag{1}$$

The envelope of these spherical sound waves, that is the shock front, is found at any particular time (t = Constant) by solving equation (1) simultaneously with its derivative with respect to the trajectory parameter τ:

$$XX' + YY' + ZZ' = -a^2(t - \tau) = -aR \tag{2}$$

At any particular time, the sound wave fronts and their envelopes are fixed in space, and equations (31), page 65, of reference 10 are therefore applicable. Use of those equations in the present context yields
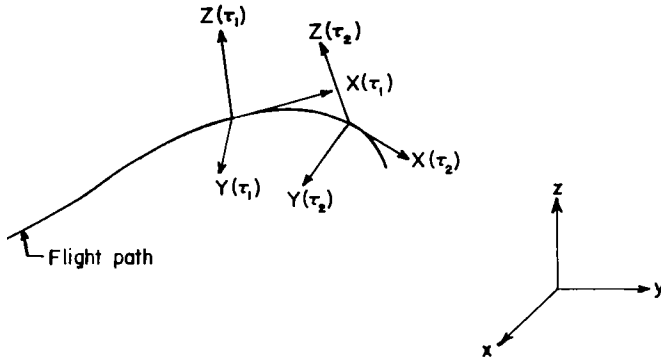

Figure 1.- Moving-trihedral coordinate system.

$$X' = V \frac{dX}{ds} = V\left(\frac{Y}{\rho} - 1\right) \qquad (3a)$$

$$Y' = V \frac{dY}{ds} = -V\left(\frac{X}{\rho} + \frac{Z}{T}\right) \qquad (3b)$$

$$Z' = V \frac{dZ}{ds} = V \frac{Y}{T} \qquad (3c)$$

where ρ is the radius of curvature of the flight path, V is the speed of flight, and T is the radius of torsion of the path. Substituting the values for X', Y', and Z' from equations (3) into equation (2) and simplifying gives

$$-VX = -aR \qquad (4)$$

or

$$X(\tau) = \frac{R}{M} \qquad (4a)$$

The characteristic line corresponding to any given value of τ is therefore a circle lying in a plane which is perpendicular (at the center of the circle) to the instantaneous tangent to the flight path at time τ, and which is at a distance R/M from the position of the airplane at time τ. The radius of the circle $\sqrt{Y^2 + Z^2} = R\sqrt{1 - \frac{1}{M^2}}$ measures the extent of the spreading of the shock front. At any fixed value of t, the set of all these circles associated with different values of τ forms the wave envelope, or shock front. If sections of the envelope tend to overlap, extreme overpressures may be generated along the line of intersection, which is mathematically an edge of regression (ref. 10, p. 60) and is called the cuspidal ridge or cusp line in the sonic-boom literature. This line is found by differentiating equation (4) with respect to τ:

$$V'X + VX' = V'X + V^2\left(\frac{Y}{\rho} - 1\right) = -a^2$$

or

$$V'X + \left(\frac{V^2}{\rho}\right)Y = V^2 - a^2 \qquad (5)$$

When equation (5) is solved (with respect to the parameter τ), together with equations (1) and (4a), the following parametric equations of the edge of regression are obtained:

$$X_c = \frac{R}{M} \tag{6a}$$

$$Y_c = \rho\left(\lambda - \frac{V'}{V^2}\frac{R}{M}\right) \tag{6b}$$

$$Z_c = \pm\sqrt{\lambda R^2 - \rho^2\left(\lambda - \frac{V'}{V^2}\frac{R}{M}\right)^2} \tag{6c}$$

where $\lambda \equiv 1 - \frac{1}{M^2}$.

Consider first the case in which the airplane is turning but V' is zero. Then, equation (6c) becomes

$$Z_c = \pm\sqrt{\lambda R^2 - \rho^2\lambda^2}$$

From this equation, it is seen that the edge of regression, or cusp line (see fig. 2(a)), is symmetric about the instantaneous (at time τ) plane of the turn (the osculating plane), and that its intersection with this plane is not at the sound source, that is, not at the airplane, but at a point in the osculating plane at a distance $R_c = \rho\sqrt{\lambda}$ from the location of the source at time $\tau = t - \frac{\rho\sqrt{\lambda}}{a}$.
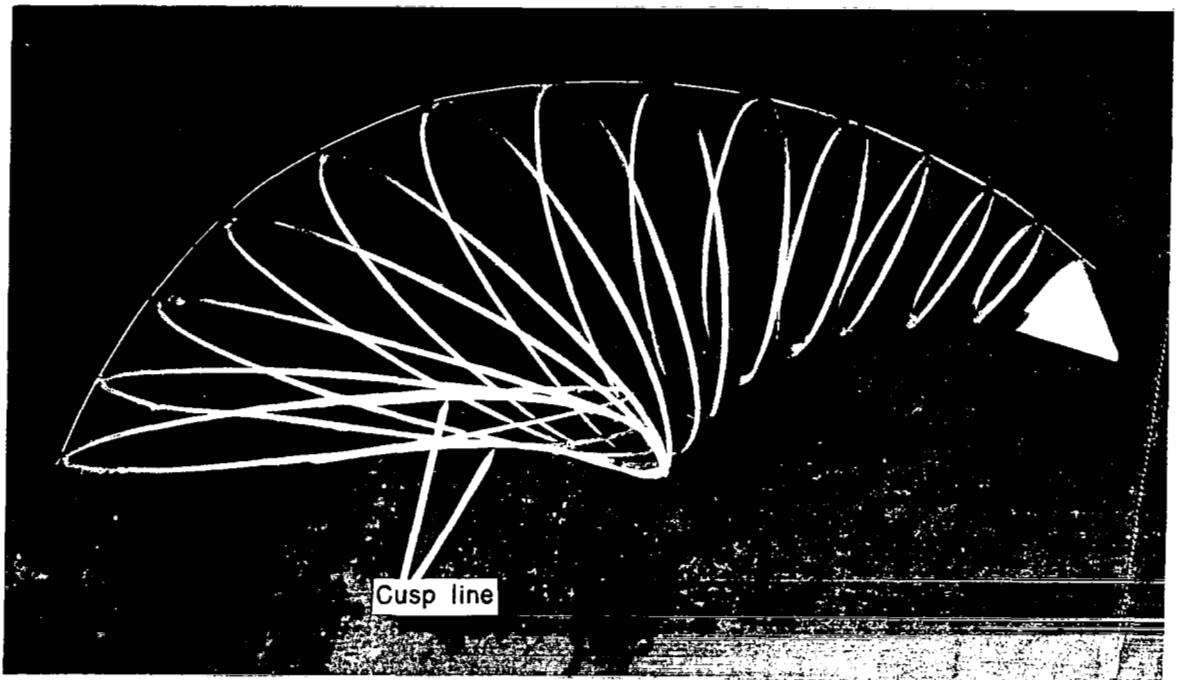
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(a) Wire model depicting cusp line and representative characteristic lines of shock envelope resulting from a planar turn.

Figure 2.- Cusp formation due to flight maneuvers.

To illustrate the application of equations (6), consider an airplane making a constant-altitude (60,000 feet) turn at a steady velocity of 2,500 ft/sec corresponding to a Mach number of about 2.5. Inasmuch as the passengers in a commercial aircraft should not be subjected to a centrifugal acceleration of more than about 0.5g, the radius of curvature of the turn must be, at least,

$$\rho = \frac{\left(2.5 \times 10^3\right)^2}{16} \approx 3.9 \times 10^5 \text{ ft}$$

or roughly 75 miles, and the cuspidal point in the osculating plane is at a distance $\rho\sqrt{\lambda}$ at a lateral distance $Y_c = \rho\lambda$ which is about 62 miles. The distance of the ground cusp point from the source would be over 63 miles. These distances are so great that in all probability these rays would never reach the ground because of atmosphere refractive effects.

Of course, it is possible for the flight-path curvature to be in the vertical rather than the horizontal plane. For example, VGH records indicate that oscillations in normal acceleration of more than ±0.5g occasionally occur in the course of commercial operations. (See ref. 11.) The data of reference 11 were taken from turbojet and turboprop transports flying at subsonic speeds, but it appears likely that a supersonic transport would be subject to similar oscillations and that these oscillations would cause variation in the boom intensity on the ground.

If, for example, an airplane at a Mach number of 1.3 (corresponding to a steady velocity of 1,300 ft/sec) is undergoing a normal-acceleration deviation of -0.5g at the crest of an oscillation, the corresponding cusp point in the vertical plane should be roughly 60,000 feet below the airplane. If the flight altitude is less than 60,000 feet, say 40,000 feet, the converging ground-track rays will not focus at ground level, but the ground-track intensity will be increased to the extent that the rays have converged.

This vertical oscillation may well account, at least in part, for the considerable variation in measured flight-track overpressures for an aircraft in nominal straight flight. (See fig. 12 of ref. 12.) The effect of this vertical oscillation should be small and possibly negligible for a supersonic transport under cruise conditions because the velocity will be so great that any curvature of the path must necessarily be small and, furthermore, because any contribution of atmospheric turbulence to this airplane flight oscillation should diminish at high altitudes. This phenomenon may prove to be significant during the ascent stage of a flight, when the velocities and altitudes are much lower than those for cruise conditions; however, any compression will be modified somewhat by the effect of the climb angle, which is to tilt the rays upward so that they travel a greater distance before reaching the ground.

When an airplane makes a turning maneuver, the part of the shock envelope on the outside of the turn will tend to spread out; thus, the intensity should be weakened. However, the predicted decrease predicated on considerations of volume effects only would not be entirely valid because some intensification due to the inclination of the airplane lift vector may occur as a result of the centrifugal acceleration.

Consider now an airplane that is accelerating but not turning. In this case, $1/\rho$ is zero, and equation (5) can then be written in the form
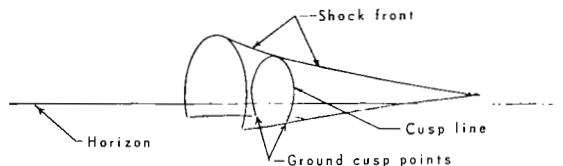
$$X = \frac{a^2}{V'}\left(M^2 - 1\right)$$

which, together with equation (6a), yields the following equation for the distance of the cuspidal ridge from the source:

$$R = a(t - \tau) = \frac{a^2}{V'} M\left(M^2 - 1\right)$$

This equation determines, at any fixed value of t, a specific value of the parameter τ for which the associated characteristic line is also the edge of regression. This cusp line is therefore a circle in a plane perpendicular to the flight path. Its radius is

$$\frac{\sqrt{M^2 - 1}}{M} R = \frac{a^2}{V'}\left(M^2 - 1\right)^{3/2}$$

Figure 2(b) is a schematic drawing of the shock wave and cusp line resulting from the acceleration of an airplane. In commercial transport flights, the acceleration V' should be limited to about 0.1g, out of consideration for passenger comfort. Therefore, when the airplane is nearing its cruise velocity, the focusing distance would be large. For example, if $M = 2$ and $a = 1,000$ ft/sec, the cuspidal ridge is a circle of radius greater than 300 miles. Under these conditions, the boom intensity would be negligible even if the focused rays did reach the earth. On the other hand, when the airplane is accelerating through the lower supersonic Mach number range, the problem is potentially a severe one. For example, for Mach numbers less than 1.25 at an altitude of 40,000 feet, the cusp points on the earth occur less than 5 miles from the ground track; that is, in the region where the intensity would already be significant, even in the absence of focusing. The superboom which may be associated with this initial acceleration into the supersonic Mach number range should be highly localized because the cusp points on the earth move rapidly away from the flight track as the Mach number increases. Therefore, after the superboom occurs on the ground track, the intensity on the ground track should decrease rapidly. An acceleration superboom has been observed experimentally. (See ref. 12.)

(b) Schematic drawing indicating formation of a cusp line resulting from aircraft acceleration.

Figure 2.- Concluded.

Formation of Shock Envelope and Cusp Line in the Presence of

a Linear Sound-Speed Gradient

In this section it will be assumed that the flight is horizontal so that the effects of refraction can be studied without the additional complication of the effect of changing altitude.

According to reference 4 (eq. (I.5)), the equation of a sound wave front in an atmosphere with a linear sound-speed gradient at time t due to a disturbance at time τ is

$$(x - \xi)^2 + (y - \eta)^2 + \left\{(z - \zeta) + \frac{a_O}{k}\Big[\cosh k(t - \tau) - 1\Big]\right\}^2 = \frac{a_O{}^2}{k^2} \sinh^2 k(t - \tau) \quad (7)$$

For constant-altitude flight, the equations can be simplified by the use of the moving-trihedral coordinate system, but even for this restricted case the analysis is correct only if the osculating plane is considered to be horizontal for straight flight. With this convention, equation (7) can be written

$$X^2 + Y^2 + \left[Z + \frac{a_O}{k}(\cosh k\tilde{t} - 1)\right]^2 = \frac{a_O{}^2}{k^2} \sinh^2 k\tilde{t} \quad (7a)$$

or

$$X^2 + Y^2 + Z^2 + 2\frac{a_O}{k}\Big(Z - \frac{a_O}{k}\Big)(\cosh k\tilde{t} - 1) = 0 \quad (7b)$$

where $\tilde{t}$ denotes $t - \tau$ and a_O denotes the sound speed at flight altitude. Taking the derivative of relation (7b) with respect to the parameter τ gives

$$XX' + YY' + ZZ' - a_O\Big(Z - \frac{a_O}{k}\Big)\sinh k\tilde{t} = 0$$

Substituting relations (3) into this equation yields

$$XV + a_O\Big(Z - \frac{a_O}{k}\Big)\sinh k\tilde{t} = 0 \quad (8)$$

or

$$X = \frac{1}{M}\Big(\frac{a_O}{k} - Z\Big)\sinh k\tilde{t} \quad (8a)$$

Equations (7b) and (8a) are the equations of the shock envelope. (A machine program for calculating the ground pattern for flight in an atmosphere with linear sound-speed gradient was developed in connection with the work of ref. 13.)

In order to find the edge of regression of the shock envelope, equation (8) can be differentiated with respect to τ and the resultant equation solved simultaneously with equations (7b) and (8a). If the flight trajectory is still assumed to be horizontal, the derivative of equation (8) with respect to τ is

$$XV' + X'V - a_0 k\left(Z - \frac{a_0}{k}\right)\cosh k\tilde{t} = 0$$

Substituting the relation for X' from equation (3a) into this equation yields

$$XV' + V^2\left(\frac{Y}{\rho} - 1\right) - a_0 k\left(Z - \frac{a_0}{k}\right)\cosh k\tilde{t} = 0$$

or

$$XV' + V^2\frac{Y}{\rho} = V^2 - a_0{}^2\cosh k\tilde{t} + a_0 kZ \cosh k\tilde{t} = 0 \qquad (9)$$

Equations (9), (8a), and (7b) are the equations of the cusp line.

The effects of refraction can be studied more clearly by isolating them from the effects of acceleration and turning. Thus, for steady, straight, level flight $\left(V' = \frac{1}{\rho} = 0\right)$, equation (9) becomes

$$V^2 = a_0\left(a_0 - kZ_c\right)\cosh k\tilde{t}$$

Therefore,

$$Z_c = \frac{a_0}{k} - \frac{V^2}{a_0 k}\,\text{sech } k\tilde{t}$$

or

$$Z_c = \frac{a_0}{k}\left(1 - M^2\text{sech } k\tilde{t}\right) \qquad (10a)$$

Substituting into equation (8a) yields

$$X_c = \frac{V}{k}\tanh k\tilde{t} = \frac{a_0}{k} M \tanh k\tilde{t} \qquad (10b)$$

which solved together with equations (7b) and (10a) gives

$$Y_c = \pm\frac{a_0}{k}\sqrt{\left(M^2 - 1\right)\left(1 - M^2\text{sech}^2 k\tilde{t}\right)} \qquad (10c)$$

Equations (10) are the parametric equations for the cusp line caused by refraction. It will be shown in the following section that for straight flight

the cusp normally does not occur at ground level, except on the ground track when the rays are just tangent to the ground.

On the ground track, $\text{sech } \tilde{k t} = 1/M$ since $Y_c = 0$. Inasmuch as a linear sound-speed gradient rarely extends beyond an altitude of 40,000 feet, the limiting Mach number at which the ground cusp will occur is found by inserting $Z_c = -40,000$ feet and $\text{sech } \tilde{k t} = 1/M$ into equation (10a) and solving for M. This limiting Mach number is thereby found to be about 1.16.

It is rather interesting that a sound wave front propagating outward in an atmosphere with a linear sound-speed gradient maintains a spherical form (see eq. (7a)) although the rays are curved. The center of the sphere moves downward with time, and the radius does not increase at a constant rate but with an acceleration. The result is a shock envelope resembling somewhat that shown schematically in figure 3.
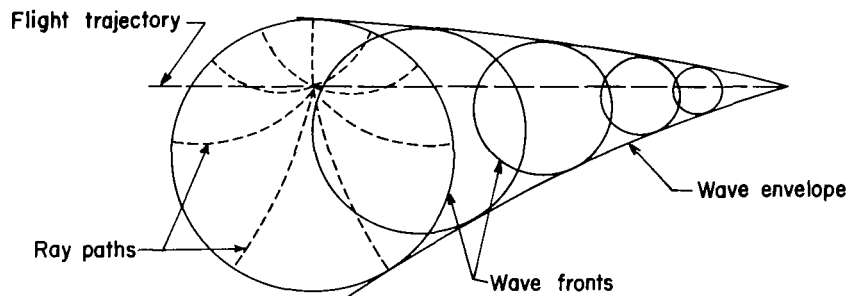

Figure 3.- Exaggerated schematic diagram of wave propagation from source moving in straight, steady, level flight in an atmosphere with a linear variation of sound speed with altitude.

Lateral Distribution of Ground-Level Boom Intensity

In the previous sections, the spatial distribution of the shock envelope and of superboom regions was investigated by means of a geometrical study of the wave-front locations. For the purpose of studying the quantitative aspects of the compression of the shock, however, it is simpler to treat the trajectories of the elements of the wave fronts, that is, the rays. One reason for this shift in point of view is the fact that Snell's law of refraction for a stratified atmosphere with no wind

$$a \sec \theta = c = \text{Constant}$$

applies directly to the rays. Furthermore, the energy flux in a region bounded by specific rays (a ray tube) remains approximately constant; consequently, the boom intensity varies inversely as the square root of the cross-sectional area of the ray tube. (See ref. 2.)

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To determine the lateral distribution of intensity in a nonuniform atmosphere by computing the ray-tube cross-sectional area is a complex problem, because the cross section cannot be taken to be a rectangle. In order to simplify the analysis, a linear vertical sound-speed gradient will be assumed, with no wind. In this atmosphere, the sound rays are arcs of circles whose centers are at altitude a_g/k. (See ref. 7.) Furthermore, inasmuch as there is no horizontal component of the sound-speed gradient, each ray remains in the same vertical plane throughout its trajectory. The range of a ray that strikes the ground is simply the length of its projection on the ground. The airplane is assumed to be in straight, steady, level flight in the x-direction. The ray tube to be examined, as shown in figure 4, consists of a part of the ray cone emitted during time Δt. This ray tube is at an angle Ω from the horizontal measured in a plane perpendicular to the flight path.
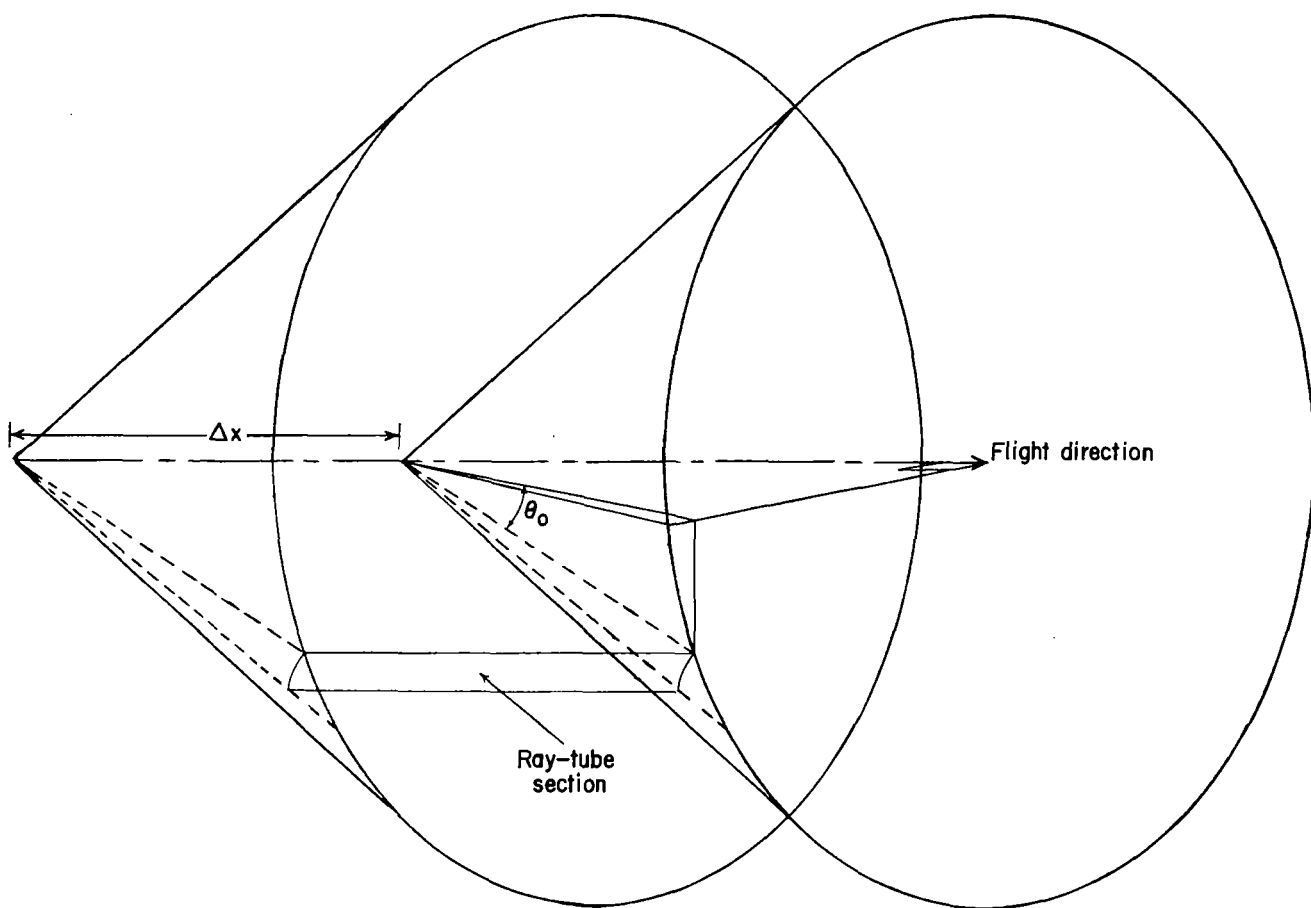
(a) Initial ray tube.

Figure 4.- Ray-tube geometry.
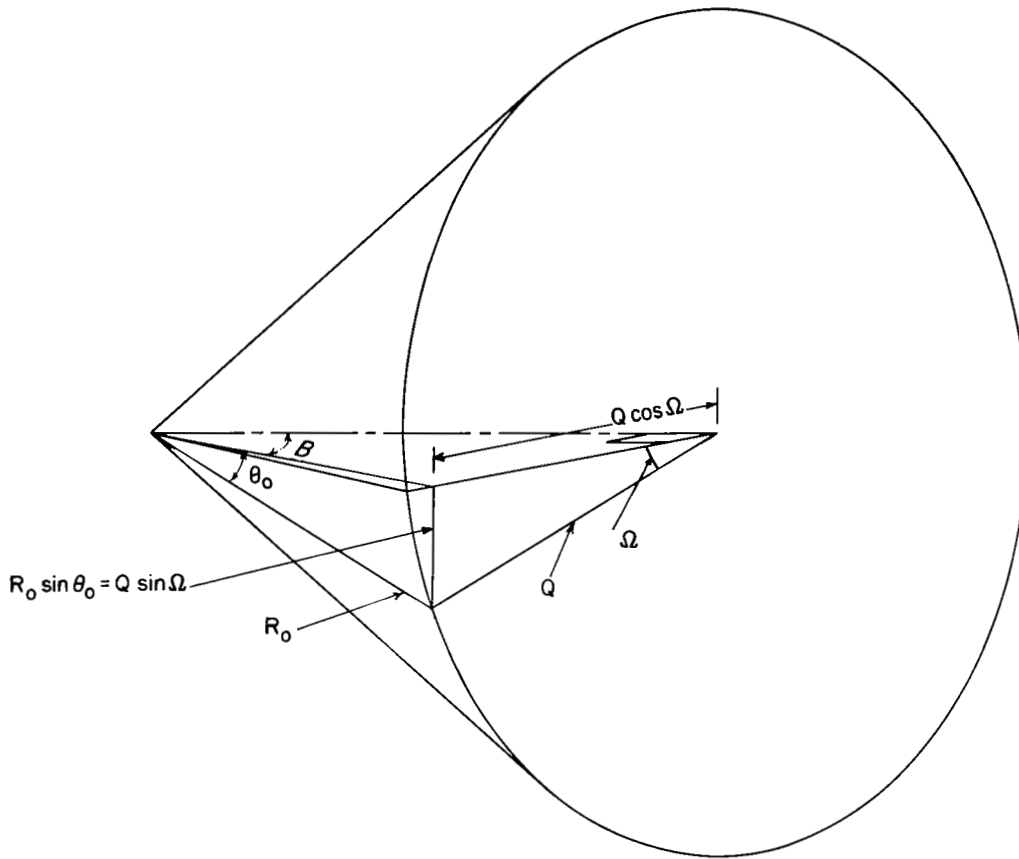
(b) Initial ray cone.

Figure 4.- Concluded.

The rays representing the upper boundary of this ray tube all have initial inclination angle θ_0. They therefore have the same inclination at each altitude and maintain the same horizontal displacement. Their ground intersection thus forms a straight-line segment of length Δx parallel to the x-direction. Similarly, the rays of the lower boundary of the ray tube all have initial inclination angle $\theta_0 + \Delta\theta_0$, and their ground strike forms a straight-line segment also of length Δx parallel to the x-direction but displaced somewhat from the ground intersection of the upper boundary rays. If the "sides" of the ray-tube ground intersection are approximated as straight lines, this intersection is a parallelogram with base length Δx. The "height" of the parallelogram is the incremental component of the range Δy_g, normal to the x-direction, of the rays associated with the incremental angle $\Delta\Omega$.

In order to calculate this parallelogram "height," an expression for y_g as a function of Ω is needed. A ray associated with the angle Ω and originating at altitude h has initial inclination θ_0 and radius of curvature
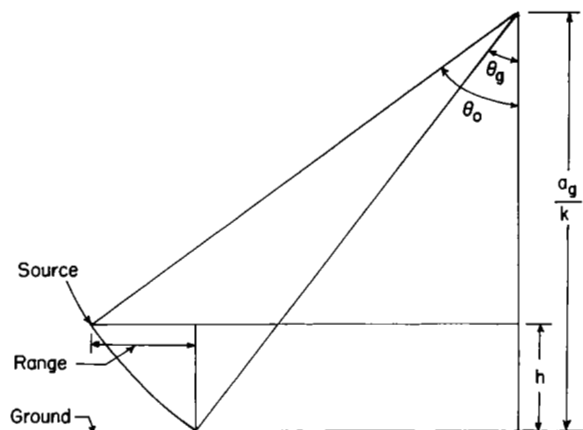
Figure 5.- Sketch used for calculating range of a sound ray in an atmosphere with a linear sound-speed gradient.

$\left(\dfrac{a_g}{k} - h\right)\sec\theta_O$. The radius of curvature can also be expressed as $\dfrac{a_g}{k}\sec\theta_g$; thus, θ_g and θ_O are related by the following expression:

$$\sec\theta_g = \left(1 - \frac{kh}{a_g}\right)\sec\theta_O \equiv H\sec\theta_O \qquad (11)$$

The range of the sound ray is $\left(\dfrac{a_g}{k} - h\right)\tan\theta_O - \dfrac{a_g}{k}\tan\theta_g$. (See fig. 5.) If the vertical plane containing the ray forms an angle B with the vertical plane through the ground track, the y-component of the range is

$$y_g = \sin B\left[\left(\frac{a_g}{k} - h\right)\tan\theta_O - \frac{a_g}{k}\tan\theta_g\right] \qquad (12)$$

Figure 4(b) represents conditions existing in the immediate neighborhood of the source of the disturbance, and the lengths R_O and Q are therefore infinitesimal quantities. Within this small region, the rays are virtually straight lines. A relationship that is apparent from this figure is

$$Q\cos\Omega\csc B = Q\sin\Omega\cot\theta_O$$

or

$$\sin B = \cot\Omega\tan\theta_O \qquad (13)$$

Also

$$R_O\sin\theta_O = Q\sin\Omega$$

or

$$\sin\theta_O = \sqrt{1 - \frac{1}{M^2}}\sin\Omega = \sqrt{\lambda}\sin\Omega \qquad (14)$$

Expressing θ_g, B, and θ_O in terms of Ω by means of equations (11), (13), and (14) and substituting into equation (12) yields

$$y_g = \sqrt{\lambda}H\frac{a_g}{k}\frac{\cos\Omega}{1 - \lambda\sin^2\Omega}\left(\sqrt{\lambda}\sin\Omega - \sqrt{1 + \frac{\lambda\sin^2\Omega - 1}{H^2}}\right) \qquad (15)$$

The incremental y-range is then

$$\Delta y_g = \frac{dy_g}{d\Omega}\Delta\Omega \qquad (16)$$

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The ground intersection of the ray tube has an area

$$\Delta A_g = \Delta x \ \Delta y_g$$

which can be determined with the use of equation (16). The actual ray-tube cross-sectional area ΔA is the projection of ΔA_g onto the surface tangent to the wave front. If the unit vector normal to this surface - that is, the unit tangent to the ray path - is denoted by $\vec{r}_N$, then

$$\Delta A = \vec{r}_N \cdot \vec{k} \ \Delta A_g$$

$$= \sin \theta_g \ \Delta A_g$$

$$= \sqrt{1 - \frac{\cos^2 \theta_o}{H^2}} \ \Delta A_g$$

$$= \sqrt{1 + \frac{\lambda \ \sin^2 \Omega - 1}{H^2}} \ \Delta A_g \tag{17}$$

From this equation, ΔA (and hence the intensity distribution) can be found directly as a function of Ω. The result for ΔA is

$$\Delta A = \Delta x \ \Delta \Omega \sqrt{\lambda} H \frac{a_g}{k} \sqrt{1 + \frac{\lambda \ \sin^2 \Omega - 1}{H^2}} \left\{ \left[\frac{\lambda \ \sin 2\Omega \ \cos \Omega}{\left(1 - \lambda \ \sin^2 \Omega\right)^2} - \frac{\sin \Omega}{1 - \lambda \ \sin^2 \Omega} \right] \left(\sqrt{\lambda} \ \sin \Omega - \sqrt{1 + \frac{\lambda \ \sin^2 \Omega - 1}{H^2}} \right) \right.$$

$$\left. + \frac{\cos \Omega}{1 - \lambda \ \sin^2 \Omega} \left(\sqrt{\lambda} \ \cos \Omega - \frac{\lambda \ \sin 2\Omega}{2H^2 \sqrt{1 + \frac{\lambda \ \sin^2 \Omega - 1}{H^2}}} \right) \right\} \tag{17a}$$

The limiting value of y_g occurs when

$$1 + \frac{\lambda \ \sin^2 \Omega - 1}{H^2} = 0$$

in equation (15). This limiting y_g-range, which is

$$y_{g,max} = \lambda H \frac{a_g}{k} \frac{\cos \Omega \ \sin \Omega}{1 - \lambda \ \sin^2 \Omega}$$

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applies to those rays that are just tangent to the ground, at the lateral edge of the ground pattern. The ray-tube area at the extremity of the ground pattern is, from equation (17a),

$$\lim_{y_g \to y_{g,max}} \Delta A = -\Delta x \; \Delta \Omega \; \frac{\lambda^{3/2}}{2H} \; \frac{a_g}{k} \; \frac{\cos \Omega \sin 2\Omega}{1 - \lambda \sin^2 \Omega}$$

This quantity is positive because $\Delta \Omega$ is negative, and it cannot be zero except on the ground track ($\Omega = 90°$). Inasmuch as a cusp point corresponds to the convergence of a ray tube to zero cross-sectional area, it follows that a cusp point resulting solely from the refractive effect of a linear sound-speed gradient can occur at the ground only on the ground track.

As y_g increases (with constant $\Delta \Omega$), Δy_g increases and therefore ΔA_g in equation (17) increases but $\sin \theta_g$ decreases. When the ground pattern is only a few miles wide, Δy_g is still relatively small even at the lateral edge of the ground pattern, and $\sin \theta_g$ decreases so rapidly for rays off the flight track that the net result may be a smaller ray-tube area off the flight track than on it. In other words, the intensity may be higher off the ground track than on it. This situation is illustrated by some results that have been calculated by means of equations (15) and (17a). (See fig. 6.)
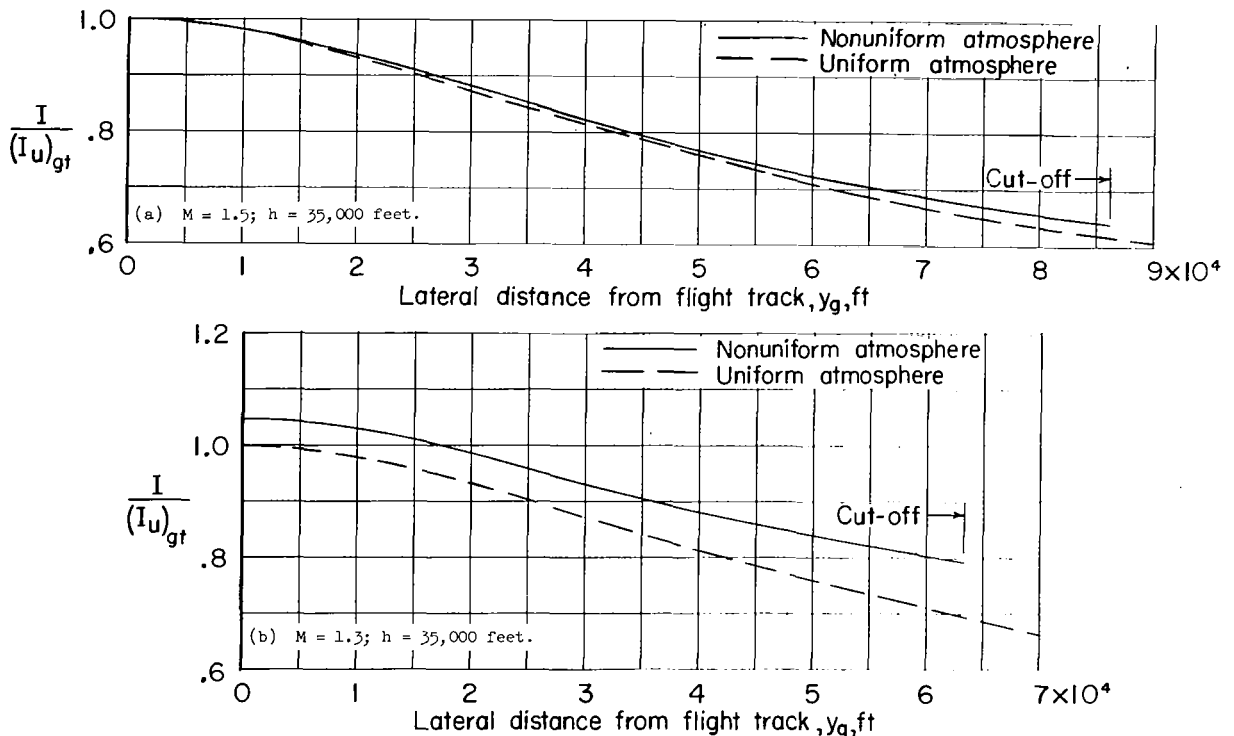

Figure 6.- Several representative plots comparing lateral distribution of intensity on ground for flight in a uniform atmosphere with that for flight in an atmosphere with a linear sound speed.
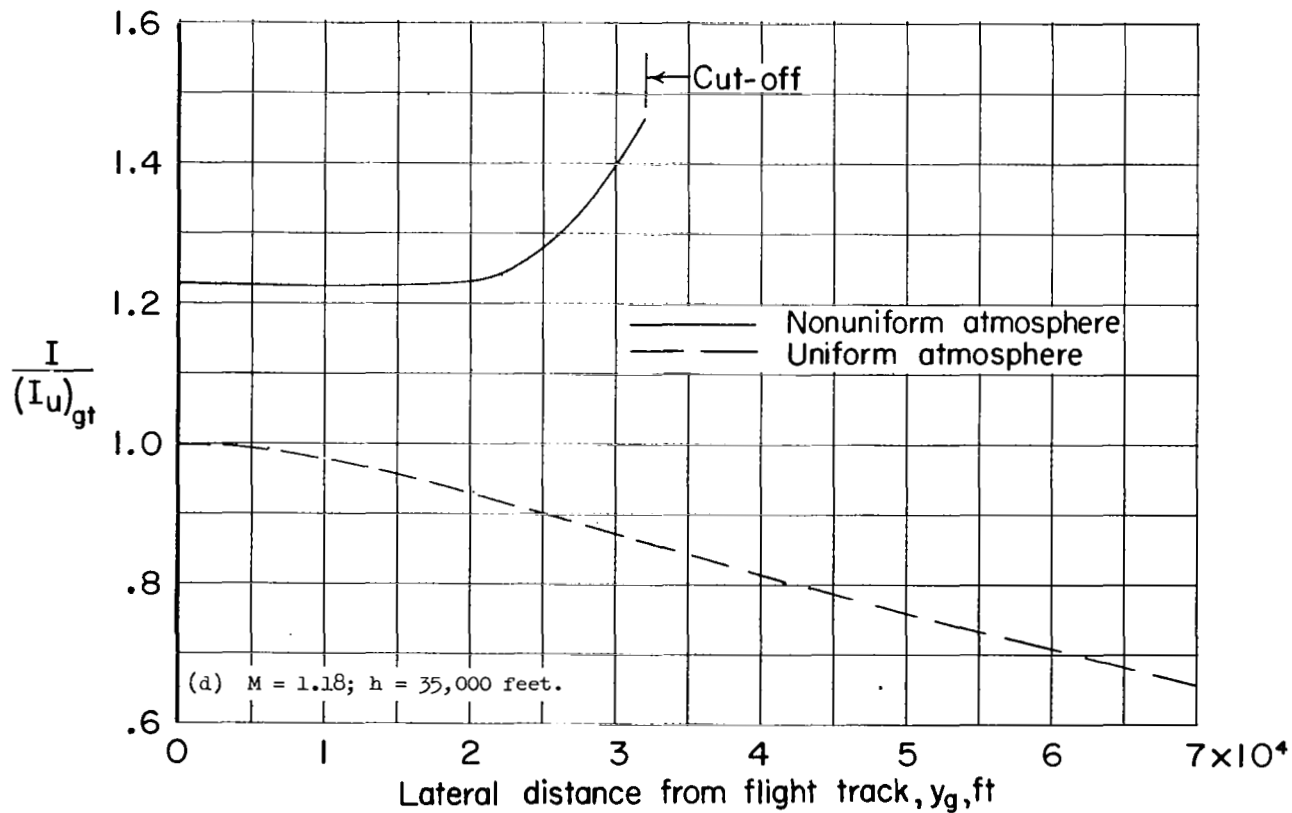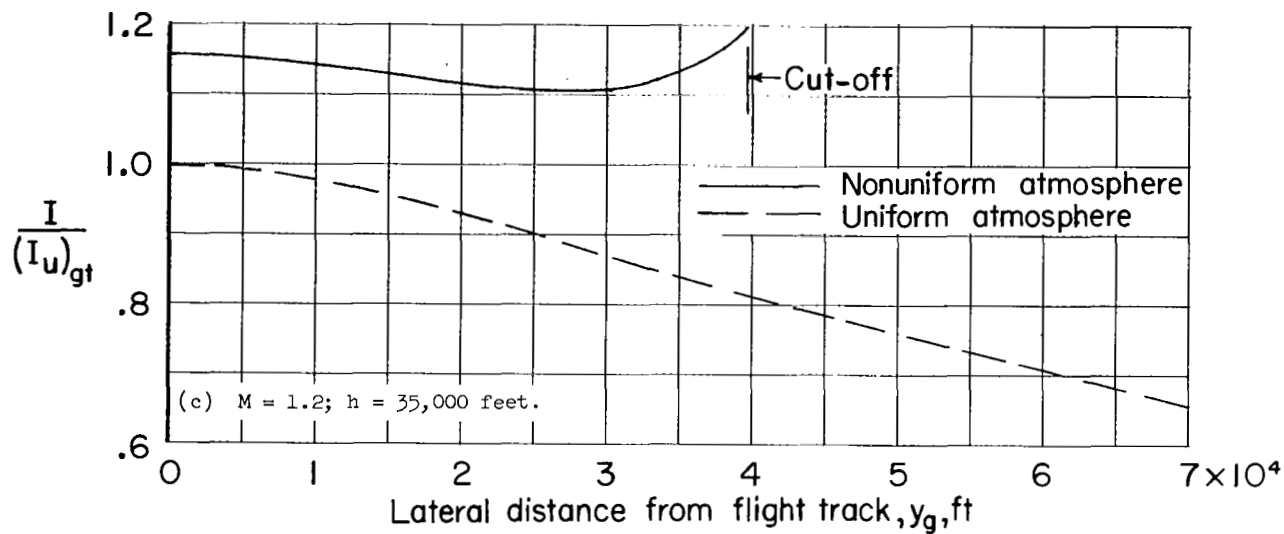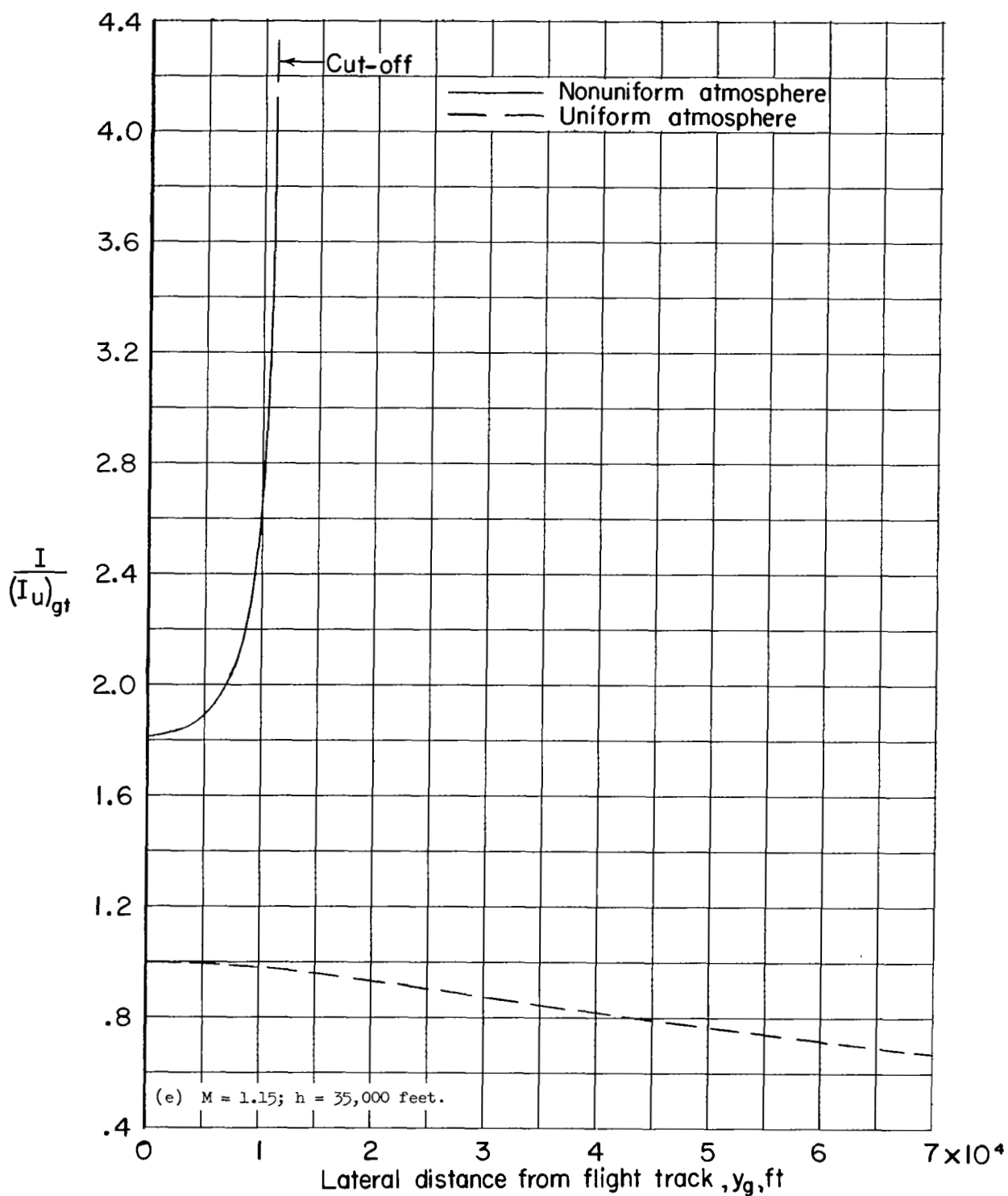
(c) M = 1.2; h = 35,000 feet.

(d) M = 1.18; h = 35,000 feet.

Figure 6.- Continued.

Figure 6.- Continued.

(f) M = 1.2; h = 25,000 feet.
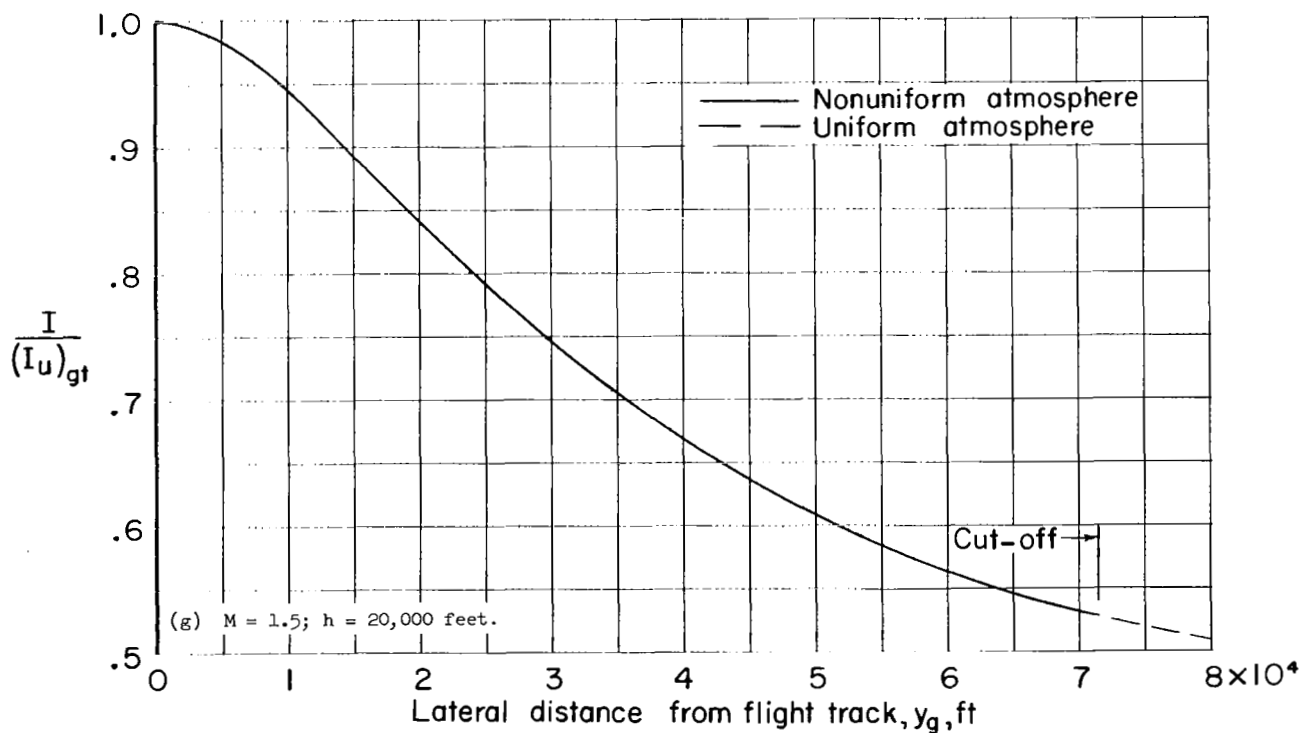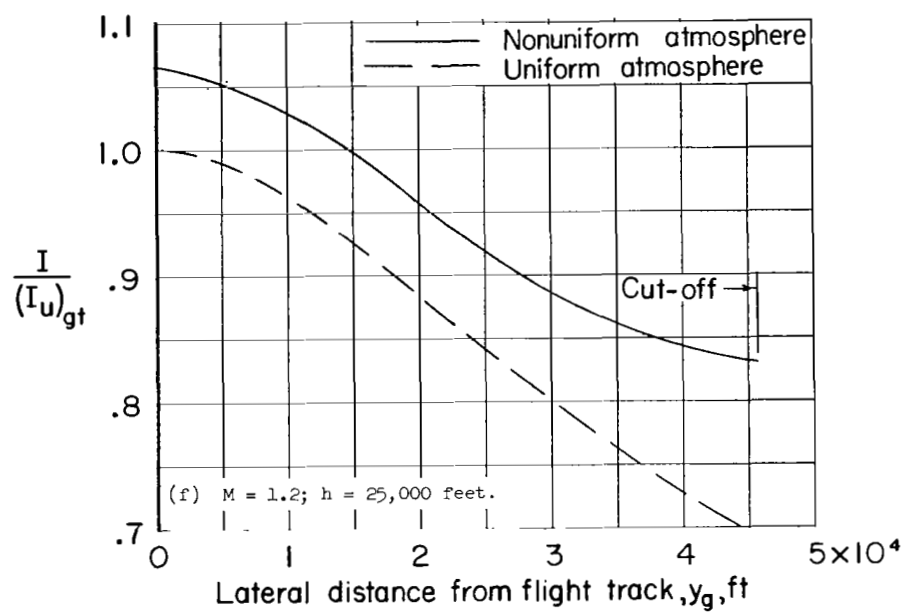

(g) M = 1.5; h = 20,000 feet.

Figure 6.- Concluded.

Figures 6(a) and 6(g) indicate that, in general, for $M \gtrsim 1.5$, the boom intensity can be calculated with good accuracy without considering the bending of the rays that results from the temperature gradient. Two considerations should not be neglected in relating these theoretical results to the actual physical situation. First, the rays near the lateral edge of the ground pattern are nearly tangent to the ground and thus are more subject to the influence of atmospheric turbulence near the ground than are the steeper rays. The effect of this turbulence is generally to distort the pressure signature of the wave. (See ref. 12.) A second important consideration is the obvious one that the atmosphere at any given time will not conform precisely to the stationary, linear-sound-speed-gradient model assumed in this analysis. At times the actual gradient is stronger, and under some conditions the local gradient near the ground is sharp. Under such conditions, the effects of refraction would be important at higher Mach numbers than indicated in this analysis.

Intensity of Ground-Track Rays in a General Stratified Atmosphere

If the atmosphere is assumed to be stratified - that is, if all gradients are assumed to be in the vertical direction - and if the winds are parallel to the vertical plane including the wave-front normal (for ground-track rays this vertical plane includes the x-axis), the effect of the wind can be accounted for in the law of refraction as follows (see, for example, ref. 7):

$$c = a(z) \, \sec \, \theta(z) + u(z) \tag{18}$$

In other words, equation (18) is applicable when the ray is subjected to horizontal headwinds and tailwinds but not crosswinds. When crosswinds must be accounted for, the more general analysis of reference 8 can be used. This general analysis yields the law of refraction in the form of a pair of equations which, in the present notation and with the neglect of any vertical component of the wind velocity, become

$$\left. \begin{aligned} c_1 &= a \, \sec \, \alpha + u + v \, \cos \, \beta \, \sec \, \alpha \\ c_2 &= a \, \sec \, \beta + u \, \cos \, \alpha \, \sec \, \beta + v \end{aligned} \right\} \tag{19}$$

where c_1 and c_2 are constants for the ray path.

In theory, equations (19) provide the basis for a general analysis of the effect of an arbitrary vertical variation of wind and temperature on an arbitrary section of the shock wave. However, the complexity of this kind of analysis would not be justified for the purposes of the present study, which are to investigate the nature of the effects and to assess their order of magnitude. Inasmuch as maximum wind effects occur when the full wind component is parallel to the vertical plane containing the wave-front normal, these effects can be accounted for by equation (18) and this form of the law of refraction will therefore be used hereafter in this analysis. The letter u will denote the refracting wind which is zero at flight altitudes. In order to account for the wind at flight altitude, u can be replaced by $u - u_0$ and u_g by $u_g - u_0$ in the resulting equations; but, in this case, dx/dt will denote the airplane speed plus the wind speed at flight altitude.

20

Consider two ground track rays, horizontally displaced a distance $\Delta x = V \Delta \tau$, emitted from an airplane flying in the x-direction at altitude h. (See fig. 7.)

Figure 7.- Geometry of ground-track ray propagation in presence of wind and temperature gradients.

Inasmuch as both rays have initially the same inclination, they both have the same characteristic velocity. Furthermore, because the medium is stratified, none of the factors affecting the distance traveled in the x-direction are functions of x. Thus, at each level, both rays are subjected to the same wind and temperature gradients, and the horizontal displacement Δx remains constant. The perpendicular distance Δd between the rays is $\Delta x \sin X$, where X is the angle that the rays make with the horizontal:

$$X = \text{arc} \tan\left(-\frac{dz}{dx}\right) = \text{arc} \tan \frac{(-dz/dt)}{dx/dt} = \text{arc} \tan \frac{a \sin \theta}{a \cos \theta + u}$$

Then,

$$\Delta d = \Delta x \sin X = \Delta x \frac{a \sin \theta}{\sqrt{a^2 + 2au \cos \theta + u^2}}$$

The quantity Δd represents approximately the height of a ray tube that is symmetric with respect to the vertical plane containing the ground track (at $\Omega \approx 90°$) (figs. 7 and 4(b)), and it is clear that this height can approach zero if the ray paths are tangent to the earth. The width of this tube is approximately represented by

$$-\Delta \psi \int_h^Z \csc \theta \, dz$$

and its cross-sectional area at the ground is therefore

$$\Delta A_g = -\Delta \psi \ \Delta x \ \frac{a_g \ \sin \theta_g}{\sqrt{a_g^2 + 2a_g u_g \ \cos \theta_g + u_g^2}} \int_h^0 \csc \theta(z) \ dz \qquad (20)$$

Inasmuch as $\theta(z)$ is a rather complicated function of the wind and sound-speed distribution, it is desirable to approximate this expression by a simpler function of u and a. Such an approximation can be made by assuming that u/a is much smaller than unity. This assumption is normally in accord with physical fact.

First, $\csc \theta$ in equation (20) can be rewritten as

$$\csc \theta = \frac{\sec \theta}{\sqrt{\sec^2 \theta - 1}} = \frac{\sec \theta}{\sqrt{\sec \theta + 1}\sqrt{\sec \theta - 1}}$$

Then, by means of equation (18), in the form

$$\sec \theta = \frac{a_o \ \sec \theta_o}{a} - \frac{u}{a}$$

the variable θ can be replaced in the equation for $\csc \theta$, and thus in the equation for ΔA_g (eq. (20)), by the variables a and u:

$$\Delta A_g = -\Delta \psi \ \Delta x \ \frac{\sin \theta_g}{\sqrt{1 + 2\dfrac{u_g}{a_g} + \dfrac{u_g^2}{a_g^2}}} \int_h^0 \frac{\dfrac{a_o \ \sec \theta_o}{a} - \dfrac{u}{a}}{\sqrt{\dfrac{a_o \ \sec \theta_o}{a} - \dfrac{u}{a} + 1}\sqrt{\dfrac{a_o \ \sec \theta_o}{a} - \dfrac{u}{a} - 1}} \ dz$$

In approximating this expression, one can neglect quantities of the order u/a or smaller, except in the factor $\sqrt{\dfrac{a_o \ \sec \theta_o}{a} - \dfrac{u}{a} - 1}$ which, for slightly inclined rays, is quite small and, since it occurs in the denominator, has the effect of making the entire expression sensitive to both wind and sound-speed gradients. The resultant approximation is

$$\Delta A_g \approx -\Delta \psi \ \Delta x \ a_o \ \sec \theta_o \ \sin \theta_g \int_h^0 \frac{dz}{\sqrt{a_o \ \sec \theta_o + a(z)}\sqrt{a_o \ \sec \theta_o - a(z) - u(z)}}$$

where $\theta_g = \text{arc sec}\left(\dfrac{a_o}{a_g} \sec \theta_o - \dfrac{u_g}{a_g}\right)$.

The ray-tube cross-sectional area for an airplane flying in a uniform atmosphere without wind is as follows:

$$\Delta A_u = \Delta x \, \sin \theta_0 \, \Delta\psi \, \csc \theta_0 \, h = \Delta x \, \Delta\psi \, h$$

The ratio of the intensities is then

$$\frac{I}{I_u} = \left(\frac{\Delta A_u}{\Delta A}\right)^{1/2} = \left[\frac{h}{a_0 \, \sec \theta_0 \, \sin \theta_g \displaystyle\int_0^h \frac{dz}{\sqrt{a_0 \, \sec \theta_0 + a(z)}\sqrt{a_0 \, \sec \theta_0 - a(z) - u(z)}}}\right]^{1/2}$$

(21)

For $a_0 = 970$ ft/sec, headwind $u_0 = 100$ ft/sec, $h = 36,000$ feet, $u = 60$ ft/sec, wind calm at ground level, and $a_g = 1,115$ ft/sec, then the increase in intensity due to the refractive effect of wind and temperature gradients as calculated from equation (21) is well below 10 percent for Mach numbers of 2 or higher.

For the important case of linear sound-speed gradient $\left(a = a_g - kz\right)$ occurring simultaneously with a linear wind gradient $\left(u = u_g - lz\right)$, the denominator of the integrand in equation (21) can be expressed as the square root of a quadratic function of z, and the integral can be evaluated in closed form. (See formula 165 of ref. 14.)

In general, for a supersonic transport flying at or near cruise conditions with atmospheric conditions that may reasonably be expected, the refractive compression of sound waves should be less than 10 percent. On the other hand, when the airplane accelerates through the low supersonic Mach number range, it will pass through a critical Mach number such that the ground-track rays are just tangent to the ground, if the atmosphere is such as to cause upward bending of the rays. Inasmuch as the cross-sectional area of the ray tube approaches zero under these conditions, the theory predicts a superboom at the point of tangency. Thus, the theory predicts a significant refractive compression of flight-track rays for the general Mach number range in which the compression due to acceleration is also considerable. Moreover, it is in this same range (low supersonic) that considerable curvature of the flight path due to vertical oscillations is possible, which possibility adds another mechanism that may augment the boom intensity.

If the gradient of the tailwind is sufficient to cause downward bending of the rays, the height of the ray tube will increase, since Δx is constant over the path, and the boom intensity should therefore be somewhat lower because of the refraction.

In the absence of wind, the curvature of a sound ray can be found readily with the use of the refraction equation (18) in the following form:

$$a(z) \, \sec \theta(z) = c$$

Differentiating this relation gives

$$-\frac{da}{dz} \sec \theta - a \sec \theta \tan \theta \frac{d\theta}{dz} = 0$$

$$\frac{1}{\rho} = \frac{d\theta}{dr} = -\frac{\cos \theta}{a} \frac{da}{dz}$$

Thus, the less the inclination of a ray the greater is its curvature.

Now, if an airplane is in straight, steady, level flight, two ground-track rays emitted a distance Δx apart will appear as in figure 7. Both rays have the same inclination at flight altitude, and indeed at each successive level, and therefore their actual perpendicular separation distance must decrease as they bend outward.

If the airplane is accelerating, the first of two reference rays emitted will have less inclination and hence greater curvature than the second. This differential curvature causes the rays to converge more rapidly than they would if they were straight, as in a uniform atmosphere. The effect of the refraction is therefore to cause the acceleration cusp below the airplane to occur closer to the position of the source (when the cusp rays were emitted) than it would in a uniform atmosphere.

If the airplane is decelerating, the first of the reference rays has more inclination and consequently less curvature than the second, and, therefore, spreading of the rays results.

Relative Effects of Wind and Temperature Gradients

It may be of interest to obtain an estimate of the relative effects of wind and temperature by calculating the conditions under which the temperature and wind refraction exactly cancel so that there is no net bending of a sound ray.

If the ray (assumed to be in the xz-plane) experiences no net refraction, its inclination angle X remains constant. Therefore,

$$\frac{d}{dz} \cot X = 0 \tag{22}$$

Now

$$\cot X = \frac{dx/dt}{dz/dt} = \frac{a \cos \theta + u}{a \sin \theta} = \cot \theta + \frac{u}{a} \csc \theta$$

where again u denotes the refracting wind and so can be replaced by $u - u_0$ to account for the wind at flight altitude. Then, equation (22) becomes

$$-\left(\csc^2\theta + \frac{u}{a} \csc \theta \cot \theta\right)\frac{d\theta}{dz} + \frac{\csc \theta}{a} \frac{du}{dz} - \frac{u}{a^2} \csc \theta \frac{da}{dz} = 0 \tag{23}$$

24

The quantity $d\theta/dz$ can be expressed in terms of du/dz and da/dz by differentiating equation (18) and solving for $d\theta/dz$:

$$\frac{d\theta}{dz} = - \frac{\cot\theta \frac{da}{dz} + \cos\theta \cot\theta \frac{du}{dz}}{a} \tag{24}$$

Substituting equation (24) into equation (23) and rearranging yields

$$\frac{du}{dz} = - \frac{\cos\theta + \frac{u}{a}\cos 2\theta}{1 + \frac{u}{a}\cos^3\theta} \frac{da}{dz} \tag{25}$$

The angle θ could be expressed in terms of u and a by equation (18), but the form of equation (25) is satisfactory for a qualitative discussion. From this equation one can see that for those parts of the shock wave front that are nearly vertical ($\theta \approx 0^\circ$), the wind gradient required to prevent upward bending of the rays has approximately the same magnitude as the sound-speed gradient; but, for a part of the envelope that is nearly horizontal ($\theta \approx 90^\circ$), the required gradient is only about u/a of the sound-speed gradient. In terms of ground-track rays, equation (25) indicates that the wind gradient has relatively more refractive effect at higher Mach numbers. At a Mach number of 2, the wind gradient that would exactly counteract the effect of the temperature gradient would be only about one-half of the value required at a barely supersonic Mach number.

Mechanisms of Focusing by Winds and Ground Structures

It is a consequence of Fermat's principle (the ray path is such as to make the ray passage time an extremum) that initially divergent rays which descend over their entire paths through a stratified medium cannot converge. (See ref. 7.) However, machine calculations of ray paths indicate that with certain wind and temperature distributions (ref. 6), a cusp line parallel to the flight track can occur, and it may be of some interest to describe here the mechanism of such a focusing.

Consider two rays of the ray cone that are very near the horizontal. Then, as an approximation, they can be considered to lie in the same vertical plane. Suppose, now, that the wind gradient is such as to oppose the bending effect of the negative sound-speed gradient and, also, that the wind gradient is so great that a downward bending of the rays occurs. If one of the rays has a slight initial upward inclination, it receives more support from the higher wind speed at the greater altitude and thus may arrive at some point on the ground at the same time as the lower ray, as illustrated in figure 8. Refractive effects of this type have been observed
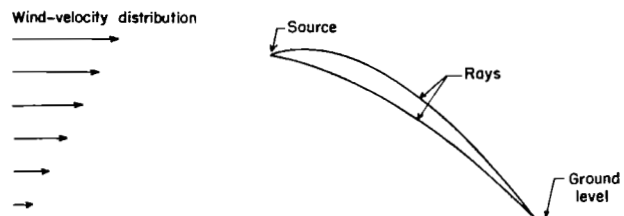

Figure 8.- Diagram illustrating mechanism by which two initially diverging rays can converge at ground level.

experimentally at low altitudes when the temperature gradient is slight. (See ref. 15.)

Similar effects may occur for flight in other regions where the temperature gradient is slight but where the wind gradient is strong. Such conditions often occur in the altitude range extending roughly from 35,000 feet to 50,000 feet. In this range, the temperature gradient is usually slight, and the winds are often decreasing in magnitude with altitude in such a way that a strong wind gradient exists.

In the prior sections, three possible causes of shock compression have been discussed: aircraft acceleration, trajectory curvature, and atmospheric refraction. Another possibility is that focusing may occur because of the manner in which the rays are reflected from nonplanar terrain or structures on the ground. Inasmuch as the shock wave near the ground may be treated locally as a plane wave, and the rays as straight lines, the geometric calculation of the ray-tube area in this case is straightforward.

One possibility of this kind of focusing is illustrated in figure 9 where a ray tube is shown being reflected off the adjacent walls of two perpendicular structures. If the angle of the ray tube is symmetric with respect to these two walls, the reflected ray tube will tend to compress toward the vertical plane that bisects the angle of the walls. This region of compression may intersect the ground or, possibly, the skylight of a lower building.
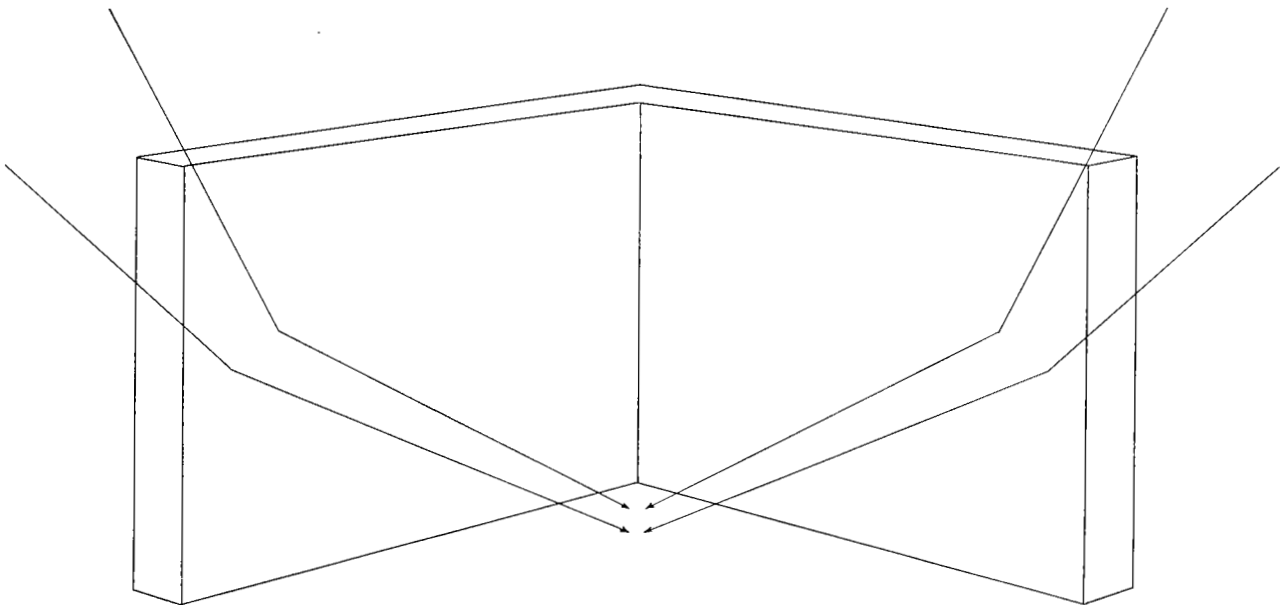

Figure 9.- Schematic drawing illustrating the focusing of a ray tube as a result of reflection from the walls of a building with two perpendicular wings.

CONCLUDING REMARKS

At the altitudes and Mach numbers of interest in connection with flights of a supersonic transport at or near cruise conditions, superboom effects due to acceleration, turns, or atmospheric refraction should be negligible. On the other hand, in the low supersonic range, all these effects, in addition to that of changing altitude, may influence the boom intensity.

As the Mach number increases, the wind becomes relatively more important in refracting the ground-track rays. Focusing of ground-track rays (which are initially parallel) can be caused by the temperature gradient, or by a combination of wind and temperature gradients. Cusp points off the ground track cannot be caused by a linear sound-speed gradient, but when the combination of flight altitude and Mach number is such that the ground pattern is relatively narrow, then it is possible that the intensity off the flight track will be greater than on it. Focusing of slightly inclined rays at a considerable distance from the flight track can occur in the presence of a strong wind gradient. Focusing may also occur as a result of reflection from certain forms of terrain or ground structures.

Equations are presented for calculating the influence of wind and sound-speed gradients on the boom intensity of ground-track rays and for calculating the lateral distribution of intensity in the presence of a linear sound-speed gradient.

Langley Research Center,
    National Aeronautics and Space Administration,
        Langley Station, Hampton, Va., October 30, 1963.

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